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PARACHUTES FOR AIRCRAFT

By Waldemar Müller

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Air Forces and Opening Shock

In order to provide the essential data on air resistance, rapidity of opening, shock stresses in opening, etc., for subsequent use, we will first give a few test results.

In October, 1918, parachute descents were made at Döberitz from airplanes having an air speed of about 40 m/s (131 ft./sec., or 89.5 mi./hr.). The parachutes had a somewhat flattened hemispherical shape with a diameter $d = 5.4$ m (17.7 ft.) and a projected area $F = 22.9\ m^2$ (246.5 sq.ft.). The results are given in Table I, where

H = altitude of airplane above the ground,

G = weight of load including harness,

t = time from release to opening of parachute,

T = total time of descent from airplane to ground,

P = maximum resistance while opening,

V = mean speed of descent.

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*"Fallschirme für Luftfahrzeuge," "Zeitschrift für Flugtechnik und Motorluftschiffahrt," October 28, 1927, pp. 470-481.

TABLE I.

	Special characteristics	H m	G kg	t sec.	T sec.	P kg	V m/s	Stability
1	None	450	200	2.5	54	450	8.4	Good
2	With 50 m trip line	450	200	3	50	900	8.6	"
3	" 1 shroud line removed	400	100	3	90	300	4.3	"
4	With 3 shroud lines removed	400	100	3	60	150	6.5	–
5	With 20 holes of 80 mm diameter near top, to prevent oscillations	500	100	3	102	150	4.8	Poor
6	With 500 mm central vent	400	100	3	80	250	4.8	"
7	Porous muslin	400	100	3	–	–	–	Good
8	Muslin	400	100	3	102	200	3.8	–
9	"	450	100	3	88	250	4.4	–
10	Muslin with shroud lines shortened, to prevent oscillations	400	79	3	99	320	3.9	Good
11	Muslin, 5 lines broken, one tear of 2 m and one of 1 m. Parachute opened smoothly	400	100	3	102	350	3.8	"

The descents were made on different days and consequently under various weather conditions, which fact accounts for the great differences under otherwise like conditions. If we should calculate the resistance values c from these results according to the formula

$$W_F = \text{total weight} = c\,\gamma\,\frac{v^2}{2g}\,F = G$$

in which γ = air density in kg/m^3 and g = acceleration due to gravity = 9.81 m/s^2, we would obtain values varying between 1.5 and 4.7. Aside from the fact that the difference between the separate values is so great that no probably correct value can be obtained, even the minimum value c = 1.5 seems very large. The speeds of descent appear, therefore, to have been greatly affected by eddies and ascending air currents.

Even the maximum stresses show great differences, varying, for example, between 150 and 350 kg (330-770 lb.) for a 100 kg (220 lb.) load. These differences cannot be laid to ascending air currents, because the shock of opening is produced only by the relative motion of the parachute with respect to the air. Since the airplane speeds and the air densities did not vary excessively, the variations in the maximum stresses must be ascribed, in general, to irregularities in the opening of the parachutes.

The total opening time was 2.5-3.0 seconds. As shown by the observations, the parachute, which was at first towed along with the airplane by the trip line and hence in advance of the parachutist for about half a second, corresponding to a distance of one to two parachute lengths, was bent back in the flight path behind the parachutist. During the next 2 seconds the parachute remained almost closed and had the appearance of a long hose. Only during the third second it suddenly opened wide with an audible report. If it is assumed, in the abovementioned cases, in which the horizontal initial speed was about 40 m/s

(131 ft./sec.), that the mean speed of the parachute, descending obliquely through the air, was 30 m (98 ft.) per second for 2 seconds after bending in the flight path, it would then correspond to a distance of about 60 m (197 ft.) through the air. The rest of the distance required for complete opening can be estimated at about 15 m (49 ft.), which makes the total distance about 75 m (246 ft.), not including the 1 - 2 parachute lengths required for the bending of the parachute path. Aside from irregularities, as we shall see later, this distance is constant for a given parachute and is independent of the speed of the parachute while opening. The opening time is therefore dependent on the speed. If the length of the shroud lines, about 10 m (33 ft.), is added to the total opening distance, then about 90 m (295 ft.) is the minimum altitude at which this parachute can be used to descend vertically from a stationary balloon, without any bending of the parachute path. On the other hand, an altitude of only about 50 m (164 ft.), corresponding to a falling time of about 3 seconds is necessary for descending from an airplane flying horizontally at 40 m (131 ft.) per second.

Table II gives the results of descents of very flat parachutes from a captive balloon. These experiments were performed at Reinickendorf in April, 1918. The parachutes were even flatter than the one shown in Figure 1, but probably assumed about this shape while descending. The diameter of the individual parachutes was $d = 8$ m (26 ft.) and their projected area was

$F = 50.27\ m^2$ (541 sq.ft.), while the basket parachute had a diameter of 13 m (43 ft.) and a projected area of $F = 132.73\ m^2$ (1429 sq.ft.). In Table II, t_L = time taken by the last 20 m (66 ft.) of descent and V_L = the mean descending speed during that time.

Otherwise the same conditions prevail as in Table I.

TABLE II.

	Type of parachute	H m	G kg	t sec	T sec	V m/s	t_L sec	V_L m/s	$\frac{V_L}{V}$
1	Individual	300	80	1.5	65	4.6	5.0	4.0	0.87
2	"	300	80	1.5	64	4.6	5.0	4.0	0.87
3	"	300	80	1.5	64	4.6	4.9	4.1	0.89
4	"	300	80	2.0	63	4.7	5.0	4.0	0.85
5	"	300	80	2.3	66	4.5	4.7	4.2	0.93
6	"	250	80	2.0	64	3.9	4.2	4.8	1.23
7	Basket	200	120	3.0	64	3.1	7.0	2.9	0.94
8	"	300	120	1.8	72	4.1	6.0	3.3	0.80
9	"	300	120	3.0	65	4.6	5.8	3.5	0.76
10	"	300	200	4.8	75	4.0	–	–	–

The descents were made on different days. The resistance coefficients, if based on the diameter according to the drawing, would fluctuate between 1.2 and 1.7 for the individual parachutes and between 0.7 and 1.5 for the basket parachute, which was geometrically similar. Since, however, the parachutes were actually flatter than the one shown in the drawing and were considerably

contracted by the pull of the shroud lines, their projected area while descending was considerably smaller than represented in the drawing. The resistance coefficients would be correspondingly higher. On account of the great discrepancies between the individual values, no probably correct values can be deduced from these speeds of descent.

The mean opening time for the individual parachute was about 2 seconds. This indicates a descent of about 10 m (33 ft.) on the assumption of an approximately free fall during the first 1.5 seconds. The further opening distance for the last half-second can be estimated at about 5 m (16 ft.). If 12 m (39 ft.) is added for the length of the shroud lines, we have a minimum altitude of about 30 m (98 ft.), at which this parachute can be used. The short opening distance of only 15 m (49 ft.) is due to the flatness of the parachute.

The total opening time for the basket parachute averaged about 3 seconds. If we assume 2 seconds to be the duration of the approximately free fall, we have a fall of about 17 m (56 ft.), and the further fall during the last second can be estimated at 8 m (26 ft.), thus making a total opening distance of 25 m (82 ft.). With 20 m (66 ft.) shroud lines, the operating altitude for this parachute would therefore be about 50 m (164 ft.). As will be shown later, the increase in the opening distance, as compared with the similar individual parachute, is approximately proportional to the increase in the linear dimensions.

Table II also shows that the landing speed V_L is smaller than the speed of descent V. (In experiment No. 6, where this was not the case, there were evidently unusual disturbances.) This lessening of the speed of descent was due to the fact that the air between the ground and the parachute was compelled to flow off to the side in a more obtuse angle than during the descent in free air.

According to Table II, the mean ratio of the landing speed to the descending speed was about 0.85. If we consider, however, that the speeds of descent (due to upward air currents) generally appear too small and that, on the other hand, the actual landing speed is even smaller than the mean value of the last 20 m (66 ft.) of descent, we can accordingly estimate the mean ratio V_L/V at less than 0.8%. This speed ratio is probably approximately the same for all parachutes, since the ratio of the length of the shroud lines (or distance of the parachute skirt from the ground at the instant of landing) to the diameter of the parachute is about the same for all parachutes.

For a hemispherical shell with the concave side turned toward the wind, with a diameter $d = 0.25$ m (0.82 ft.), a wind velocity $v = 10-12$ m (33-39 ft.) per second and an index value $vd = 2.5-3.0$ m^2 (27-32 sq.ft.) per second, Eiffel obtained a resistance coefficient $c = 1.328$. This value is considerably smaller than the resistance coefficients, which would be obtained from Table I. Table II would also yield greater values in most

cases, if the calculation of the resistance coefficient were based on the actually smaller projected area of the parachute during its descent.

On the one hand, parachutes are now made somewhat shallower and also held by the central shroud line during the descent somewhat flatter than the surface of a sphere. On the other hand, the parachute has a vent, which, under certain conditions, might increase the resistance by a "nozzle effect." The resistance coefficient c for a parachute model with d = 0.3 m (0.98 ft.) diameter and 0.13 m (0.43 ft.) height and with different vents, was therefore determined in the 0.9 m (2.95 ft.) wind tunnel of the German aeronautic laboratory in Adlershof. It was so arranged that, in all the experiments, a constant weight P = 5 kg (11 lb.) had to be supported by the air resistance. The results are assembled in Table III. The values for the 10 mm (.394 in.) vent are mean values for 22 experiments, while the other values each represent a single experiment.

TABLE III.

Diameter of vent in mm	0	10	15	20
Pressure in air stream in mm Hg = b	757.2	753.5	753.5	753.5
Absolute temperature in air stream in °C = T	283.9	283.9	284.5	285.0
Air density = $\gamma = 1.293 \frac{273}{760} \frac{b}{T}$ in kg/m³	1.240	1.232	1.230	1.223
Dynamic pressure q measured by Pitot tube in mm water column = kg/m² min.		52.6		
mean	53.8	53.6	55.0	53.3
max.		55.7		
Resistance coefficient $c = \frac{P}{F_q} = \frac{5}{0.0707\ q}$	1.315	1.320	1.285	1.325
Air velocity $v = \sqrt{\frac{2g\ q}{\gamma}}$ in m/s	29.15	29.20	29.60	29.20
Index value vd in m²/s	8.75	8.76	8.88	8.76

Table III shows that the resistance coefficients c are practically the same for the different vents, namely, about 1.32. The differences fall within the error limits, as shown by a comparison of the dynamic pressures for the different vents with the minimum and maximum pressures for the 10 mm (.394 in.) vent. The errors are chiefly due to the oscillations of the water column in the pressure gauge, which amount to several millimeters at the air velocity employed. Measurements of the static pressure near the vent likewise yielded no noteworthy pressure differences. It may therefore be concluded that the customary vents are so small in comparison with the total sail area, that they do

not appreciably affect the air resistance of the parachute.

If it is also considered that Eiffel found values for c from 1.08 to 1.23 for flat circular plates of 0.0707 - 0.5 m^2 (.761 - 5.38 sq.ft.) surface area, for which the larger values were mostly obtained by falling tests from the Eiffel tower, it may be assumed that the resistance coefficients would not vary much, even for parachutes shallower than the one represented by the tested model.

a) The Opening or Inflation Diagram in Connection with the Derivation of the Similarity Ratios.

In order to obtain an idea of the nature and magnitude of the opening shock, we may first understand the process of opening. If a person jumps from an airplane with a parachute, the latter remains apparently closed for about two seconds after leaving the pack and has the appearance of an extended hose. It then usually opens in a fraction of a second with an audible report. During the first period of about two seconds, the parachute opening is so small that very little air can enter. As soon, however, as there is a noticeable opening (about 1/50 of the projection area of the fully opened parachute) the further opening occurs very quickly. Even during the latter period, however, the parachute travels quite a distance. During this period the parachute is subjected to the greatest stresses, which are more liable to cause a rupture, the greater the velocity of the airplane and the

greater the load suspended from the parachute.

The forms assumed by a parachute while opening are diagrammatically represented in Figure 2. During the first period A, the parachute assumes a hose-like shape with many longitudinal folds extending from the skirt to the vertex, which begins to take the shape of an inflated roll. This phenomenon is explained by the fact that the air entering the parachute can at first continue to flow unimpeded, without converting its momentum into pressure. The shape A then changes gradually into the shape B, and finally into the approximately hemispherical shape C, in which only the attachment points of the shroud lines to the skirt are drawn slightly inward.

Of course all the irregularities which develop during the opening, cannot be followed mathematically. It will therefore be assumed, in what follows, that the shroud lines have no contracting effect on the skirt, an assumption which is permissible, because of the small temporary ratio of the diameter of the opening to the length of the shroud lines. It will be further assumed that the air pressure on the sail area at any instant is uniform from the skirt to the vertex and that there is no vent nor central shroud line. During the process of opening, the longitudinal folds in the vicinity of the skirt will be very numerous, very small, and entirely uniform so that a cross section through the parachute at any point will be assumed to be a circle. The fully opened sail will be assumed to have the shape of the

surface of a hemisphere with the radius r. The parachute will be assumed to open in the form of a cylinder capped by a hemispherical shell, which gradually enlarges, during the opening, to the final form with the radius r, while the cylinder gradually grows broader and shorter until it vanishes entirely.

The opening of the parachute can therefore be regarded as the inflation of a hemispherical balloon with increasing cross section of the inflation opening dependent on the capacity of the parachute at the time. Figure 3 represents the parachute during and at the close of the process of opening. Let

I_1 = the momentary capacity of the hemispherical shell in cubic meters;

I_2 = the capacity of the underlying cylinder in cubic meters;

I = $I_1 + I_2$ in cubic meters;

n = radius of momentary hemisphere and cylinder in meters;

F = momentary cross section of cylinder in square meters;

s = path of parachute through the air in the direction of the parachute axis in meters.

Since the distance from the parachute peak to the edge of the skirt is $r \frac{\pi}{2}$ and the distance from the parachute peak to the annexed cylinder is $n \frac{\pi}{2}$, the height of the cylinder is $(r - n) \frac{\pi}{2}$. The following formulas then hold good.

$$F = n^2 \pi; \quad n = \sqrt{\frac{F}{\pi}} \tag{1}$$

$$I = I_1 + I_2 = \frac{2}{3} n^3 \pi + r n^2 \frac{\pi^2}{2} - n^3 \frac{\pi^2}{2} \tag{2}$$

If equation (1) is now inserted, we obtain

$$I = I_1 + I_2 = F r \frac{\pi}{2} - \sqrt{F^3} \left(\frac{\sqrt{\pi}}{2} - \frac{2}{3\sqrt{\pi}} \right)$$

or, in another form,

$$I = F r \frac{\pi}{2} - 0.51 \sqrt{F^3} = f(F) \tag{3}$$

The curve $I = f(F)$ can now be drawn, as is done in Figure 4 for a hemispherical parachute with radius $r = 2.9$. By inverting the dependence ratio, we obtain

$$F = \varphi(I) \tag{4}$$

By calculating the reciprocal values of F we obtain the function

$$\frac{1}{F} = \frac{1}{\varphi(I)} = \psi(I) \tag{5}$$

This serves as an auxiliary function for calculating the $s = \Phi(I)$ curve whose conditions from Figure 3 give

$$\left. \begin{aligned} \Delta I &= F \, \Delta s \quad \text{or} \quad d\,I = F\,ds \\ ds &= \frac{1}{F}\, dI = \psi(I)\, dI \\ s &= \int \psi(I)\, dI = \Phi(I) \end{aligned} \right\} \tag{6}$$

The final value of s is the total opening distance. This is independent of the velocity, as follows from the equation. From

With an initial opening of $0\ m^2$, the distance s would therefore be infinitely great. As is easily seen, however, there is always a slight preliminary opening, due to the thickness and stiffness of the parachute fabric. This gives the opening distance a finite value and the greater the initial opening is, the shorter the opening distance will be.

In Figure 4 the graphic integration is begun with $F = 0.5\ m^2$ corresponding to $I = 2\ m^3$, it being assumed that the parachute had opened to this cross section during the first two seconds. For this two-second period (which is naturally greater for larger similar parachutes, corresponding to the total opening distance, and smaller for like parachutes with greater speed), the stresses, accelerations, etc. can be only approximately calculated, because the course of the opening process during this period depends largely on the manner of packing the parachute, on the individual structural parts and on accidental contingencies. In subsequent calculations this period is designated by the letter A.

Since the opening distance s, as follows from the above equations and from Figure 4 has, for a given parachute, a definite final value, independent of the speed, which final value depends only on the shape of the parachute, such an inflation diagram is of great value for judging a parachute as to its suitability for descending from a swiftly moving airplane or from a low altitude.

The following formulas apply to similar parachutes with the linear similarity ratio λ, if the values F', I', and s' in the similar parachute correspond to the values F, I, and s:

For the original parachute,

$$I = f(F) \tag{3}$$

$$F = \varphi(I) \tag{4}$$

$$ds = \frac{1}{F}\, d\, I \tag{6}$$

$$s = \int \frac{1}{F}\, d\, I = \Phi(I)$$

$$s = \psi(F) \tag{7}$$

For the similar parachute,

$$I' = \lambda^3\, I = \lambda^3\, f(F) \tag{3a}$$

$$F' = \lambda^2\, F = \lambda^2\, \varphi(I) \tag{4a}$$

$$ds' = \frac{1}{F'}\, d\, I' = \frac{1}{\lambda^2\, F}\, d(\lambda^3\, I) = \lambda \frac{1}{F}\, d\, I \tag{6a}$$

$$s' = \lambda \int \frac{1}{F}\, d\, I = \lambda\, \Phi(I) = \lambda\, s$$

$$s' = \lambda\, s = \lambda\, \psi(F) \tag{7a}$$

For similar parachutes with the linear similarity ratio λ, the opening processes are geometrically similar and the opening cross sections are proportional to λ^2, the volumes proportional to λ^3 and the opening distances proportional to λ, i.e., to the diameters.

The inflation diagram (Fig. 4) gives for the hemispherical model parachute with the radius $r = 2.9$ m (9.5 ft.) from $F = 0.5\ m^2$ (5.38 sq.ft.) to complete opening or inflation, a

total opening distance $s = 11.2$ m (36.7 ft.). The inflation diagram for the similar parachute (Fig. 5) with the similarity ratio $\lambda = 2$ gives from $F = 2$ m² (21.53 sq.ft.) up to complete inflation, an opening distance $s' = 22.4$ m² (241 sq.ft.), i.e. $s' = 2$ s.

b) Stresses Developed During the Opening in Connection with the Derivation of the Similarity Ratios.

With the aid of the inflation diagram, which gives the parachute volume as plotted against the opening cross section, the air resistance developed at any point of the opening distance can be calculated, provided the resistance coefficient, corresponding to the shape of the parachute at the time, and the momentary speed are known.

The air-resistance coefficient of a fully opened approximately hemispherical parachute was found by experiment to be about $c = 1.32$, as given above, with an index value of $d =$ about 88 m² (947 sq.ft.) per second. If a parachute of about 5.8 m (19 ft.) diameter, loaded with one man, be assumed to have a descending speed of 6 m (19.7 ft.) per second in still air, the corresponding index value for the same would be $v\ d =$ about 35 m² (377 sq.ft.) per second. If it be further assumed that, on leaving the airplane, the speed of the parachute at the instant of completed opening is about twice as large as the speed of descent, corresponding to a force equal to four times the weight

of the man, we then have an index value of d = about 70 m^2 (753 sq.ft.) per second. The question now arises as to whether, with the increase of the index value from 8.8 to 70 m^2/s (94.7 to 753 sq.ft./sec.), any change, especially any diminution of the resistance coefficient as, e.g., has been observed with slender round objects, is to be expected.

According to Prandtl (Cf. Fuchs-Hopf, "Aerodynamik," 1922, p. 186 ff.), it is to be assumed that the change in the resistance coefficient is due to the fact that, with increase in speed, the original laminar flow becomes turbulent, even before the separation point is reached. The thin wedge of still air behind the separation point is then washed away again and the separation point moves farther back, where a new state of equilibrium is established. Consequently the vortex field behind the object grows smaller, thereby lessening the resistance. In the present case it may be assumed that the separation point coincides with the edge of the fully opened parachute and that a diminution of the resistance coefficient, at least for the fully opened parachute, is not to be expected at all speeds with an increase in the index value. During the process of opening, however, a diminution of the resistance coefficient, on the basis of the cylindrical unfolding, according to Figure 2, would probably be possible, as likewise with the assumption of the actual relations corresponding to a more pear-shaped inflation. Then the greater part of the air, coming in contact with the parachute opening, flows into

the parachute and the edge of the parachute does not have a separating effect. Nevertheless, the actual process of opening develops so many irregular folds which increase the resistance, that we can calculate with a constant resistance coefficient, especially as numerical accuracy is not essential. Hence, it will be assumed that the resistance coefficient of the parachute has a constant value $c = 1.32$.

γ = air density in kg/m³.

g = acceleration due to gravity = 9.81 m/sec.².

$\frac{\gamma}{2g} = \frac{1}{16}$ = constant.

c_f = 0.8 = constant = resistance coefficient of parachutist lying obliquely to the direction of motion. (No experimental results are available on this point. For comparison, it may be remarked that, according to the Göttingen experiments, the coefficient of a cylinder of 0.3 m (0.98 ft.) diameter, at speeds of 15-21 m (49-69 ft.) per second, falls from 1.2 to 0.3 and then slowly rises again.

f = 0.5 m² (5.38 sq.ft.) = resistance area of parachutist lying obliquely to the direction of motion.

W_F = momentary resistance of parachute in kg.

W_f = resistance of parachutist lying obliquely to the direction of motion.

P_o = $W_F + W_f$ = total resistance of parachute and occupant in kg whereby, as soon as the parachute exerts sufficient force to pull the parachutist out of the oblique position into a position parallel to the flight path, W_f becomes so small that it can be disregarded.

If the momentary speed v of the parachute is measured in meters per second, then

$$W_F = c \frac{\gamma}{2g} F v^2 = c \frac{1}{16} F v^2 \tag{8}$$

$$W_f = c_f \frac{\gamma}{2g} f v^2 = c_f \frac{1}{16} f v^2 \tag{9}$$

$$P_o = W_F + W_f \tag{10}$$

In the descent of a load from an airplane, the path traversed by the parachute or the load and the stresses generated can be calculated successively for each individual point of the path. The individual periods of time must thereby be chosen so small that the stresses and their directions during a time period can be regarded as constant and the magnitude of the stress equal to the initial stress in the given period and its direction in the direction of the tangent to the path at the beginning of the period.

Figure 6 represents the path of descent, which is to be calculated from the point D to the point E.

P_o = the initial stress in kg at the point D.

V_o = the initial speed in m/s at the point D.

α_o = angle of path of descent to the horizontal at the point D. P, v, and α represent the corresponding values at the end of the period at the point E, that is, after they have again been provided with the index o, they are the initial values for the next period.

x, y, and z represent the distances in meters in the x, y, and path directions during a period.

t = the time in seconds.

m = the total load, including the parachute, in kg/m × s².

The origin of the coordinates is at D. For the section D E we then have the following relations. The acceleration in the x direction equals

$$\frac{d^2 x}{d\,t^2} = - \frac{P_o}{m} \cos \alpha_o = b_x \tag{11}$$

$$\frac{d\,x}{d\,t} = - \frac{P_o}{m} \cos \alpha_o\, t + C_1 ;$$

When $t = 0$, then $\frac{dx}{dt} = v_o \cos \alpha_o$;

hence $v_o \cos \alpha_o = 0 + C_1$; $C_1 = v_o \cos \alpha_o$; and the speed in the x direction, at the end of the stretch DE, equals

$$\frac{d\,x}{d\,t} = - \frac{P_o}{m} \cos \alpha_o\, t + v_o \cos \alpha_o = v_x \tag{12}$$

$$x = - \frac{P_o}{m} \cos \alpha_o \frac{t^2}{2} + v_o \cos \alpha_o\, t + C_2 ;$$

When $t = 0$, then $x = 0$ and $C_2 = 0$. Hence the distance in the x direction equals

$$x = - \frac{P_o}{m} \cos \alpha_o \frac{t^2}{2} + v_o\, t \cos \alpha_o ; \tag{13}$$

For the y direction, in which the attraction of gravity acts, we therefore have the following equations. The acceleration in the y direction equals

$$\frac{d^2 y}{dt^2} = - \frac{P_o}{m} \sin \alpha_o + g = b_y \tag{14}$$

$$\frac{d\,y}{d\,t} = \left(- \frac{P_o}{m} \sin \alpha_o + g\right) t + C' ;$$

When $t = 0$, then $\frac{d\,y}{d\,t} = v_o \sin \alpha_o$. Hence $C' = v_o \sin \alpha_o$ and the speed in the y direction, at the end of the stretch DE, equals

$$\frac{d\,y}{d\,t} = \left(- \frac{P_o}{m} \sin \alpha_o + g\right) t + v_o \sin \alpha_o = v_y \tag{15}$$

$$y = \left(- \frac{P_o}{m} \sin \alpha_o + g\right) \frac{t^2}{2} + v_o\, t \sin \alpha_o + C'';$$

When $t = 0$, then $y = 0$. Hence $C'' = 0$ and the distance in the y direction equals

$$y = \left(- \frac{P_o}{m} \sin \alpha_o + g\right) \frac{t^2}{2} + v_o\, t \sin \alpha_o \tag{16}$$

The final speed results from the geometrical addition of $v_x + \!\!\rightarrow v_y$. Hence it is

$$v = \sqrt{v_x^2 + v_y^2} \tag{17}$$

and its direction is derived from

$$\sin \alpha = \frac{v_y}{v} \quad \text{or} \quad \cos \alpha = \frac{v_x}{v} \tag{18}$$

Likewise, on the assumption that DE is a straight line, the distance s, traversed from D to E, is derived from the equation

$$s = \sqrt{x^2 + y^2} \tag{19}$$

When the distance s, traversed by the parachute from the beginning, and the speed v are known, the stresses P_o can be calculated from equations 8-10, after the parachute cross section F, corresponding to the distance s, has been determined from the inflation diagram. Generally the speed is known only for the instant of the jump, when it has the same direction and magnitude as the airplane. We accordingly begin with the calculation of the stage AB, which covers, for the usual parachutes as shown by Tables I and II, a period of about two seconds, or, more correctly, a distance of 15-80 m (49-262 ft.), according to the size and shape of the parachute. The mean speed is estimated

for this stage, which can be only approximately calculated. For example, at an airplane speed of 45-50 m (148-164 ft.) per second, the final speed for this stage must be about 40 m (131ft.)/sec., if the assumption is correct. Otherwise this stage must be again calculated with a different assumed mean speed. Another mean cross section will then be inserted for the parachute cross section F, and likewise a mean inclination angle α. Furthermore, the resistance of the parachutist lying obliquely to the direction of motion will be considered for this stage AB, corresponding to equation (9). After we have calculated the stage AB, we can then calculate, with the final values of this stage as the initial values, a new stage BC, on the assumption of a new system of coordinates with the point B as the origin, etc. From the point B on, the parachute cross sections F, corresponding to the traversed distances s, are determined from the inflation diagram; for example, for the point B, corresponding to a distance $s = 0$, the cross section $F = 0.5$ m² (5.38 sq.ft.), etc. Beyond the point B the resistance of the parachutist W_f can be disregarded, since he has already been pulled out of his oblique position and his resistance has consequently become very small.

As regards the opening shock, the most dangerous case is the descent from a vertically falling airplane, not only because the speed of the airplane is then generally the greatest, but also because the vertical descent during the opening process is the greatest and the momentum of the falling parachutist is the most

increased by the attraction of gravity. The above equations are simplified for this special case, because $\alpha = 90^\circ$ and hence $\sin \alpha = 1$ and $\cos \alpha = 0$, so that the values of

$\frac{d^2 x}{dt^2} = b_x$, $\frac{d\ x}{d\ t} = v_x$ and x become 0, and equations (11)-(13) are eliminated.

Furthermore,

$$b_y = \frac{d^2 y}{dt^2} = - \frac{P_o}{m} + g = b \tag{20}$$

$$v_y = \frac{d\ y}{d\ t} = \left(- \frac{P_o}{m} + g \right) t + v_o = v \tag{21}$$

$$y = \left(- \frac{P_o}{m} + g \right) \frac{t^2}{2} + v_o\ t = s. \tag{22}$$

The opening process proveeds much more favorably as regards the shock, when the path during the opening of the parachute is essentially horizontal, a case which would occur principally in the use of a giant parachute for supporting a whole airplane, when the wings are not broken. In this case the acceleration due to gravity can be excluded from the calculation. Equations (14)-(16) are thus eliminated and, in equations (11)-(13), $\cos \alpha = 1 =$ constant, since $\alpha = 0$. Then

$$\frac{d^2 x}{dt^2} = - \frac{P_o}{m} = b_x = b \tag{23}$$

$$\frac{d\ x}{d\ t} = - \frac{P_o}{m} t + v_o = v_x = v \tag{24}$$

$$x = - \frac{P_o}{m} \frac{t^2}{2} + v_o\ t = s. \tag{25}$$

As stated above, the opening processes of similar parachutes are geometrically similar. Hence, Newton's general law of similarity (Cf. "Hütte" 1923, 24th edition, Vol. I, p. 402) holds good, with the elimination of the acceleration due to gravity (g), i.e. for a horizontal opening path, for every part of the opening process. The values P_o', F', m', b', v', s', t', and d' correspond to the values P_o, F, m, b, v, s, t, and the diameter d of the original parachute. Furthermore,

$$\lambda = \frac{d'}{d} = \text{the linear similarity ratio,}$$

$$\tau = \frac{t'}{t} = \text{the ratio of the corresponding time periods,}$$

$$\beta = \frac{v'}{v} = \frac{\lambda}{\tau} = \text{the ratio of the corresponding speeds.}$$

At the same air density γ/g, the ratio of the accelerated air masses equals the ratio of their volumes λ^3. If there is to be mechanical similarity, the ratio of the mass of the parachute plus the load must equal $\frac{m'}{m} = \lambda^3$. Then the ratio of the stresses will be

$$\kappa = \frac{P_o'}{P_o} = \frac{m'\ b'}{m\ b} = \frac{F'\ v'^2}{F\ v^2} = \lambda^2\ \frac{\lambda^2}{\tau^2} = \frac{\lambda^4}{\tau^2} \quad \text{or} \quad = \lambda^2\ \beta^2$$

and $$\frac{b'}{b} = \frac{\lambda}{\tau^2} \quad \text{or} \quad \frac{\beta^2}{\lambda}$$

The acceleration g comes into play in vertical or oblique opening paths. Since this is the same for the original and the similar parachute, the other accelerations, corresponding to Froude's law of similarity, must also be the same for the orig-

inal and the similar parachute at every corresponding point of their paths. It must therefore be

$$b' = b = b \frac{\lambda}{\tau^2} ;$$

hence

$$\tau = \sqrt{\lambda} = \frac{t'}{t} .$$

Then

$$\frac{v'}{v} = \beta = \frac{\lambda}{\tau} = \sqrt{\lambda}$$

and

$$\kappa = \frac{P_o'}{P_o} = \frac{\lambda^4}{\tau^2} = \lambda^3 .$$

Since $b_x' = b_x$ and $b_y' = b_y$ for every corresponding section or stage of the opening distance, the angles of inclination α' and α must also be equal for every corresponding point of the paths, and especially at their beginning. If the accelerations and angles of inclination are equal at the beginning of the corresponding stages for the original and the similar parachute, then they are also equal at the end of this stage and hence also at the beginning of the next corresponding stages.

Since these relations hold good for every period and for the corresponding system of coordinates, they also hold good for the whole opening process and therefore for the maximum force of acceleration or retardation. Moreover, since no assumptions are made regarding the course of the $s = \psi(F)$ curve or the $s' = \psi(F')$ curve, but only the similarity of the two curves is assumed according to the equations $s' = \lambda\, \psi(F)$ and $F' = \lambda^2 F$, the relations found can be transferred to any similar parachutes and to the experimental results obtained through

the support of small airplanes by parachutes, aside from the ever-present irregularities in the process of opening. Table IV gives the relations generally applicable to similar parachutes, along with a few special cases.

TABLE IV.

Model parachute	General similarity case $d'=\lambda d$; $v'=\beta v$	1 $d'=d$; $\lambda=1$ $v'=\beta v$	2 $d'=\lambda d$ $v'=v$; $\beta=1$	3 $d'=\lambda d$ $v'=\sqrt{\lambda}\,v$; $\beta=\sqrt{\lambda}$	4 $d'=\lambda d$ $v'=\lambda v$; $\beta=\lambda$
s	$s' = \lambda s$	$s' = s$	$s' = \lambda s$	$s' = \lambda s$	$s' = \lambda s$
v	$v' = \beta v$	$v' = \beta v$	$v' = v$	$v' = \sqrt{\lambda}\,v$	$v' = \lambda v$
b	$b' = \frac{\beta^2}{\lambda} b$	$b' = \beta^2 b$	$b' = \frac{1}{\lambda} b$	$b' = b$	$b' = \lambda b$
F	$F' = \lambda^2 F$	$F' = F$	$F' = \lambda^2 F$	$F' = \lambda^2 F$	$F' = \lambda^2 F$
P_o	$P_o' = \lambda^2 \beta^2 P_o$	$P_o' = \beta^2 P_o$	$P_o' = \lambda^2 P_o$	$P_o' = \lambda^3 P_o$	$P_o' = \lambda^4 P_o$
m	$m' = \lambda^3 m$	$m' = m$	$m' = \lambda^3 m$	$m' = \lambda^3 m$	$m' = \lambda^3 m$
t	$t' = \frac{\lambda}{\beta} t$	$t' = \frac{1}{\beta} t$	$t' = \lambda^3 t$	$t' = \sqrt{\lambda}\, t$	$t' = t$
Corresponding Speed of Descent					
$V = \sqrt{\frac{16\ mg}{c\ F}}$	$V' = \sqrt{\lambda}\ V$	$V' = V$	$V' = \sqrt{\lambda}\ V$	$V' = \sqrt{\lambda}\,V$	$V' = \sqrt{\lambda}\ V$

In the first special case, the parachute is like the model parachute, and only the initial speed is altered. Correspondingly, the distances s', cross sections F' and masses m' also remain unaltered.

In the second special case, the speed is the same as for the model parachute and only the parachute dimensions are altered.

In the third special case $\beta = \sqrt{\lambda}$ and correspondingly the acceleration is the same as for the model parachute.

In the fourth special case $\beta = \lambda$ and correspondingly the periods are the same as for the model parachute.

As already follows from the general similarity case, the mass m' must always be proportional to λ^3 in order to obtain similar relations. The speed of descent V', that is, the speed which the parachute would attain after an infinitely long time in falling vertically under the influences of the force of gravity and the resistance of the air, is then proportional to $\sqrt{\lambda}$.

If two similar parachutes with the linear similarity ratio λ descend from an airplane at the same angle α, and if the ratio of their masses $= \lambda^3$ and the ratio of their initial speeds is $v_o'/v_o = \sqrt{\lambda}$, then the corresponding similar periods have the ratio $\sqrt{\lambda/1}$, and in each corresponding period the air-resistance forces are proportional to λ^3, the speeds are proportional to $\sqrt{\lambda}$, and the distances are proportional to λ.

Figures 7-9 represent the calculation results for the accelerations and the corresponding speeds for horizontal and vertical opening paths for the parachutes corresponding to Figures 4-5, with the initial speeds $V_o = 50$ m/s (164 ft./sec.) and $V_o' = \sqrt{\lambda} \times 50$ m/s, as functions of the distances traversed.

Example 1.- Hemispherical model parachute with horizontal inflation diagram (Fig. 4), without gravity acceleration, braked. $r = 2.9$ m (9.5 ft.); $m = \text{kg/m} \times s^2$; $V_o = 50$ m/s. Periods $A_1 - A_2$ and $A_2 - B = 1$ s; $B - C$, etc. $= 0.1$ s. The speed of descent would be $V = 6.02$ m/s (19.75 ft./sec.) and the landing speed

would be V_L = about 4.8 m/s (15.75 ft./sec.).

Example 2.- A similar parachute with the linear similarity ratio $\lambda = 2$ and the speed ratio $\beta = 1$, that is, with the same initial speed; horizontal, without gravity acceleration, braked. $r' = \lambda\ r = 5.8$ m (19 ft.); $m' = \lambda^3\ m = 64$ kg/m × s²; $V_O = 50$ m/s (164 ft./sec.); $t' = \lambda\ t$. Periods $A_1 - A_2$ and $A_2 - B = 2$ seconds; B - C etc. = 0.2 sec.; $V' = 8.5$ m/s (28 ft./sec.); V_L' = about 6.8 m/s (22.3 ft./sec.).

Example 3.- Similar parachute with the linear similarity ratio $\lambda = 2$ and the speed ratio $\beta = \sqrt{\lambda} = \sqrt{2}$ horizontal, without gravity acceleration, braked. $r' = \lambda\ r = 5.8$ m (19 ft.); $m' = \lambda^3\ m$ = 64 kg/m × s²; $V_O' = \sqrt{\lambda} \times V_O = 70.7$ m/s (232 ft./sec.); $t' = \sqrt{\lambda} \times t$. Periods $A_1 - A_2$ and $A_2 - B = 1.414$ s', B - C, etc. = 0.1414 s'; $V' = 8.5$ m/s (28 ft./sec.); V_L' = about 6.8 m/s (22.3 ft./sec.).

Example 4.- Hemispherical model parachute. Vertical with gravity acceleration, braked. $r = 2.9$ m (9.5 ft.); $m = 8$ kg/m × s²; $V_O = 50$ m/s. Periods $A_1 - A_2 = 0.9$ s; $A_2 - B = 0.75$ s; B - C, etc. = 0.1 s; $V = 6.02$ m/s (19.75 ft./sec.); $V_L = 4.8$ m/s (15.75 ft./sec.).

Example 5.- Similar parachute with linear similarity ratio $\lambda = 2$ and the speed ratio $\beta = \sqrt{\lambda} = \sqrt{2}$ vertical with gravity acceleration, braked. $r' = \lambda\ r = 5.8$ m (19 ft.); $m' = \lambda^3\ m$ = 64 kg/m × s²; $V_O' = \sqrt{\lambda}\ V_O = 70.7$ m/s (232 ft./sec.); $t' = \sqrt{\lambda}\ t$.

Periods $A_1 - A_2 = 1.273$ s; $A_2 - B = 1.06$ s; B - C, etc. = 0.1414 s: $V' = 8.5$ m/s (28 ft./sec.); V_L' = about 6.8 m/s (22.3 ft./sec.).

Example 6.- Hemispherical model parachute, vertical, with gravity acceleration, braked. $r = 2.9$ m (9.5 ft.); $m = 4$ kg/m × s²; $V_O = 50$ m/s (164 ft./sec.). Periods $A_1 - A_2 = 0.955$ s; $A_2 - B = 0.92$ s; B - C, etc. = 0.1414 s; $V = 4.25$ m/s (13.9 ft./sec.); V_L = about 3.4 m/s (11.15 ft./sec.).

Example 7.- Similar parachute with the linear similarity ratio $\lambda = 2$ and the speed ratio $\beta = \sqrt{\lambda} = \sqrt{2}$ vertical, with gravity acceleration, braked. $r' = \lambda\ r = 5.8$ m (19 ft.); $m' = \lambda^3\ m = 32$ kg/m × s²; $V_O' = \sqrt{\lambda}\, V_O = 70.7$ m/s (232 ft./sec.); $t' = \sqrt{\lambda}\ t$. Periods $A_1 - A_2 = 1.35$ s; $A_2 - B = 1.3$ s; B - C, etc. = 0.2 s; $V' = 6.02$ m/s (19.75 ft./sec.); V_L' = about 4.8 m/s (15.75 ft./sec.).

If, in Figure 4, in the curve $I = f(F)$, the branch of $F = 0$ to $F = 0.5$ m² (5.38 sq.ft.) is assumed to be a straight line according to the equation $I = 4\ F$, then it follows, from $F = 0.01 - 0.5$ m² (.108 - 5.38 sq.ft.), that the distance $s = 4 \ln \frac{0.5}{0.01} = 4 \ln 50 = 4 \times 3.912 = 15.65$ m (51.34 ft.). At an inflow coefficient k = about 0.5 the distance would therefore be 30 - 35 m (98 - 115 ft.). In example 1, this distance was covered in the 2d second. The initial stage A is therefore resolved, in examples 1-3, into two equal stages, A_1 and A_2. For the stage A_1, in which the parachute is bent in the path of

descent and is opened to 0.01-0.04 m² (.108-43 sq.ft.), only the resistance of the parachutist and for the stage A_2 the resistance of the parachutist plus the resistance of the parachute is considered for a mean opening cross section and a mean speed. From stage B on, only the resistance of the parachute is considered. The inflow coefficient is assumed to be 0.5 for all the examples. Hence the opening distances are twice as great as shown in the inflation diagrams (Figs. 4-5).

Since, for like parachutes, the opening distance is constant at whatever angle the opening path forms with the horizontal, but the opening proceeds more rapidly in a vertical descent due to the greater speed, the periods A_1 and A_2 in examples 4-7 for vertical descent are so small that the distance traversed by the parachute is about the same as in examples 1-3.

In examples 6 and 7, as compared with examples 4 and 5, only half the load is taken as the basis, so that, for the large parachute in example 7, the same specific load (kg/m²) and consequently the same speeds of descent and landing are obtained as for the model parachute in Figure 4. The course of the speeds and accelerations, as shown also by a comparison of Figures 8 and 9, are not then in the similarity ratio to example 4 but only to example 6.

In Figures 10-12, for the comparison of the opening periods, the speeds, retardations, and distances are plotted against the time t, the values being taken from Figures 7-9. Since the periods corresponding to the individual distances in Figures 7-9

are known from the calculation, Figures 10-12 can be drawn directly.

c) Conclusions

Before conclusions can be drawn from the theoretical investigations, it must be determined as to how far the calculation corresponds to the actual process. In the calculation of the opening or inflation diagram, it was assumed that the parachute opened, as shown in Figure 3, in the form of a cylinder capped by a hemisphere, whereby the latter constantly increased, while the former constantly diminished in height until it vanished entirely. As previously mentioned, the parachute really opened as shown in Figure 2, i.e., more pear-shaped. The opening cross section F is therefore at first smaller for each volume I than assumed in the computation of the inflation diagram, while at the end of the opening process it reaches practically the same size as in the computation. The actual opening process is slower at first and the distance from one value of F to another is therefore greater than the computed value. During the rest of the process, for example, in Figure 4 from cross section F = about 5 m² (5.38 sq.ft.) on, corresponding to the form B in Figure 2, the opening proceeds faster than the inflation diagram shows, since the parachute actually has a greater content at this opening cross section than the inflation diagram shows, though the final volume is the same. In the inflation diagram, therefore, the $I = f(F)$ curve corresponding to the reality had to be bent more toward the I axis for the same

initial and final points. The $s = \psi(F)$ curve would then ascend more steeply in Figure 4 up to about $F = 5\ m^2$ (5.38 sq.ft.) and then suddenly bend in the direction of the F axis. The final value of s would then be greater than shown by the inflation diagram. This error in the diagrams is partially offset in the calculation of the stresses by the adoption of a relatively small inflow coefficient, so that the total opening distance used in the calculation corresponds approximately to the actual opening distance. Corresponding to the slower inflation at the beginning and the faster inflation at the end of the opening process, the P_o/m curve in Figures 7-12 had to be flatter at first and then steeper up to the maximum force. Likewise, the v curve descended more slowly at first and then more rapidly.

If the shapes actually assumed while opening are known through photographs, the $I = f(F)$ curve can then be drawn to correspond. If the resistance coefficients corresponding to the different shapes are also known from the experiments, then the P_o/m curve can be calculated to correspond quite accurately with the reality, by substituting the corresponding F and c for every time or distance period. The P_o/m curve would be still more accurate if, just as in section A, the mean stresses which, for example, in Figures 7-12, up to the maximum stress, average about 15% greater, were substituted for the initial stress of each section. The maximum stresses would then be smaller. These stresses were disregarded in the above calculations, since the

purpose was rather to explain the calculation method and the similarity relations, than to obtain accurate results.

In order to get an idea as to how far the computed maximum forces agree with the actual ones, they can be compared by rough calculation with the measured stresses given in Table I. Here the maximum stresses were 150-350 kg (330-770 lb.), i.e., $P_o/m = 15 - 35 \text{ m/s}^2$ in the descent of a mass $m = 10 \text{ kg/m} \times \text{s}^2$ from an airplane with a horizontal speed of $V_o = 40$ m/s (131 ft./sec.) for a parachute of 5.4 m (17.7 ft.) diameter.

Experiment 1 gave a maximum retardation of about 45 m/s² (148 ft./sec.²) with $V_o = 50$ m/s (164 ft./sec.), $m = 8 \text{ kg/m} \times \text{s}^2$ and $d = 5.8$ m (19 ft.). With $V_o = 40$ m/s (131 ft./sec.), corresponding to similarity case 1, it would therefore have given $\frac{P_o}{m} = \frac{45 \times 40^2}{50^2} = 29 \text{ m/s}^2$ (95 ft./sec.²). With a diameter $d = 5.4$ m (17.7 ft.), the maximum retardation, corresponding to the shorter opening path, would be $\frac{P_o}{m} = \frac{29 \times 5.8}{5.4} = 31 \text{ m/s}^2$ (102 ft./sec.²), a value which would be still higher in consideration of the flatter shape of the parachute used in the experiments and of the somewhat inclined path of descent. Since retardations of 15-35 m (49-115 ft.)/sec.² were measured in the above experiments, the calculation results closely approach the maximum stresses obtained in the experiments.

As the result of the comparison, it may be assumed that a calculation made according to the above assumptions gives too large rather than too small results in comparison with reality,

since the maximum stresses in the descent of a parachute can differ greatly under like conditions, due to irregularities in opening, so that it is better to calculate with too great stresses rather than with too small ones.

The above similarity laws apply directly to the actual opening process of parachutes, since the relations established for the inflation diagram in equations 3a-7a also hold good for these. Since, however, no other assumptions are made than the similarity of the inflation diagrams in the derivation of the similarity laws applicable to the stresses, these laws are also directly applicable to the actual opening processes of similar parachutes.

In practice, the most important question is the maximum stress produced in the process of opening. This can be determined by the dropping of small parachutes with a dynamometer inserted between the parachute and the load for various initial speeds and masses. The results can then be transferred to larger parachutes according to the similarity laws. Hereby the formulas for the horizontal braking are accurate enough for descents from a horizontally flying airplane or for the support of an uninjured airplane (for forced landings on unfavorable fields), since the effect of gravity acceleration is not excessive in these cases. For falls, however, only the similarity law for a vertical or oblique opening path is applicable.

The nearly vertical plunge is one of the most frequent kinds of accidents. This is much more dangerous, as shown by Figures 6-7, than the customary jumps from horizontally flying airplanes

in exhibitions. As regards strength and maximum stresses, a parachute must therefore be constructed and dimensioned with special reference to vertical descent.

The total work of retardation to any given point of the path is the integral of the $P_o/m = f(s)$ curve up to this point multiplied by the mass m. For horizontal retardation this must equal the initial kinetic energy diminished by the final energy $= \frac{1}{2}\, m\, (V_o^2 - v^2)$, hence in example 1, calculated to the speed $v = 10$ m/s $= 8/2\ (50^2 - 10^2) = 9600$ mkg. For vertical retardation it equals $\frac{1}{2}\, m\, (V_o^2 - v^2)$ plus weight of load times falling distance, hence in example 4, calculated to the speed $v = 10$ m/s $= \frac{1}{2}\, m\, (V_o^2 - v^2) + mg\,(80 + 24.9) = 9600 + 8200 = 17800$ mkg, i.e., almost twice as much as for horizontal retardation. Since the stresses at the beginning of the opening process are relatively small, it is quite comprehensible that, toward the end of the opening process in vertical retardation, the stresses increase very rapidly and in example 4 (Figs. 8 and 11) attain almost three times the stresses in example 1 (Figs. 7 and 10) for horizontal retardation, although, aside from the direction of retardatipn, the conditions are just the same.

If it is assumed that the maximum retardation which can be borne by the parachutist without injury is $P_o/m = 40$ m/s² (131 ft./sec.), hence about equal to four times the gravity acceleration, then the surface area of the portion of the P_o/m curve which lies above the parallel to the s axis at the distance

40 m/s^2, multiplied by the mass m, would represent the retardation work, which must be rendered harmless by special means, so that the value of $P_O/m = 40$ m/s^2 will not be exceeded. Since the maximum retardation is limited, this is possible for a given parachute, only by lengthening the path of the mass, by means of special intermediate links, between the first and second intersection points of the P_O/m curve with the horizontal at a distance of 40 m/s^2 from the s axis. If the parachute could be so constructed, the best way would naturally be for the P_O/m curve, from the beginning to the end of the retardation in the speed of descent, to be parallel to the s axis at a distance of 40 m/s^2 and for the maximum endurable retarding force to be exerted during the entire period of opening. The total retardation path would then be the shortest possible and the parachute could therefore be used at the shortest possible distance from the ground.

The question as to what is the maximum permissible speed at which a parachute can be used, as determined by the calculations and similarity conditions, cannot be given a general answer, even for like or similar parachutes. If the maximum endurable retardation is limited to 40 m/s^2, the stresses developed in example 2, i.e. in the large parachute with horizontal retardation, lie within the limits. On the other hand, with the same parachute and the same conditions, but with vertical retardation, the stresses would exceed the limits. As shown by the similarity re-

lations and the examples, the initial speeds and the masses per square meter of the parachute area, i.e., the sail loading for very large parachutes is considerably greater than for small similar parachutes with like retardations. Herein lies a great advantage for large parachutes, which will probably lead to the use, for swift commercial airplanes, of only giant parachutes for supporting the whole airplane and its occupants. An especial advantage of such a method resides in the fact that no one is obliged to jump from the airplane and that the operation of the parachute lies in the hands of the responsible pilot. Moreover, all the occupants together with the valuable airplane would be saved or, in case the airplane should take fire, at least the separable passenger cabin with its occupants. The greater speeds of descent and landing can, to a certain extent, be taken into the bargain, since the landing gear of the airplane absorbs a portion of the landing shock. In examples 1 and 4 the landing speed for the individual parachute was about 4.8 m/s (15.75 ft./sec.), corresponding to a free fall of about 1.2 m (not quite 4 feet). In examples 2, 3, and 5, the giant parachute had a landing speed of about 6.8 m/s (22.3 ft./sec.) corresponding to a free fall of about 2.4 m (7.9 ft.). If it is assumed that a person can safely sustain a fall of 1.2 meters, then the landing gear would likewise have to absorb a free fall of 1.2 meters, i.e., it would have to do a braking work of 1.2 m g, and in fact, with the endurable retardation of 40 m/s^2 = about 4 g.

Under the assumption of a mean retardation of 30 m/s^2 = about 3 g effected by the elasticity of the landing gear, this yields, in absorbing the shock, a cushioning travel of 1.2g/3g = 0.4 m (1.3 ft.). These values are easily attained with small airplanes and are in part exceeded by large airplanes.

If the cases satisfactorily explained by the similarity relations be disregarded, then the load (i.e., the mass) can be altered for a given parachute. If the load is reduced with the same initial speed, then the kinetic energy is also reduced. The latter will therefore be more rapidly absorbed during the opening process and the maximum stress will be diminished and brought nearer the beginning of the opening process. This will happen if, for example, the load is increased proportionally to λ^2, not to λ^3, i.e., if no greater sail loading is adopted for giant parachutes than for individual parachutes.

Example 7 shows this case in comparison with example 4, the sail loading being the same as for the individual parachute. The comparison with example 5, in which the mass is proportional to λ^3, plainly shows, especially in the graphic representations (Figs. 8-9), the shifting of the maximum stress toward the beginning of the opening process. Corresponding to half the initial energy and the somewhat shorter retardation distance to the point of maximum stress, the latter is somewhat more than half as large as in example 5. In order to attain the same maximum retardation as in examples 4 and 5, the initial energy in example

7 could be increased to almost the same value as in example 5, i.e., with the same load the initial speed V_o', in comparison with example 4, could almost equal λV_o = 100 m/s (328 ft./sec.). That the initial speed for the same sail loading must be smaller than λV_o, if the maximum retardation of the larger similar parachute is not to be greater than that of the model parachute, is necessitated by the relatively short retardation path from the beginning of the opening process to the point of maximum stress, and hence by the shifting of the maximum force toward the beginning of the opening process. For a given case of the descent of a giant parachute, one can, of course, always calculate the similar case for the model parachute. In the diagrams, example 6 thus corresponds to example 7 since, in example 6, the mass m = 4 and in example 7, therefore, $m' = \lambda^3 m$. Likewise, the speeds stand in the similarity relation $v' = \sqrt{\lambda}\, v$.

If the initial speeds are altered for the same parachute and load, with horizontal retardation, the maximum stress corresponding to the similarity case 1 (Table IV) will always occur at the same point of the opening path and be proportional to β^2. With vertical retardation, on the other hand, if the speed is increased, the maximum stress is shifted toward the beginning of the opening process, since, with the same opening distance, the air-resistance stresses, as well as the initial energy, increase in proportion to β^2, they thus mutually offset one another, as in horizontal retardation, while the additional energy $m\, g\, \Sigma\, s$

due to gravity acceleration is not increased, however, since the weight and the opening distance remain the same. The speed therefore decreases even more rapidly than the application of the first special similarity case would indicate, and the maximum stress does not increase in proportion to β^2 but in a somewhat smaller ratio. For small parachutes with the usual sail loading of 3-4 kg/m² (61-82 lb./sq.ft.), the maximum stress coincides mostly in practice with the complete opening of the parachute, and there is still a sufficient reserve of kinetic energy, so that, even when the speed is increased, there is no noticeable shift of the maximum stress.

Regarding the opening distance, it follows from the inflation diagram, that a parachute of given maximum cross section has a greater opening distance, the greater its capacity, for then a greater difference in the parachute capacity I corresponds to the difference between two parachute cross sections F, as likewise the correspondingly greater difference in the capacities corresponds to the difference between the corresponding values 1/F. The $F = \varphi(I)$ curve extends therefore in the direction I, as likewise the $1/F = \psi(I)$ curve for equal F or 1/F values. The integral $s = \Phi(I)$ is therefore greater. From this there follows a slower increase of the F values in the $s = \psi(F)$ curve and larger s sections between like F values. The kinetic energy therefore diminishes more rapidly from one parachute cross section to another, and the maximum stress decreases.

The minimum altitude for using parachutes, which, in vertical descent, corresponds to the total opening distance plus one or two parachute lengths, is greater for giant parachutes than for small similar parachutes. In vertical descent it is, however, practically independent of the load and speed, provided the initial energy of the mass is not so great that shock-absorbing or retarding devices do not have to be inserted between the parachute and the load. For an individual parachute (example 4) it is about 120 m (394 ft.). For a giant parachute (example 5) it is about 240 m (787 ft.). In the descent from horizontal flight, the minimum altitude depends on the initial speed, since the opening path becomes more nearly horizontal as the flight speed is increased. The minimum altitude for the descent from horizontal flight can be approximately calculated from the time required for complete opening of the parachute. For the model parachute this is $t = 2.9$ seconds in example 1 (Figs. 7 and 10) at 50 m/s (164 ft./sec.) initial speed. This time corresponds to a free falling distance of $h = \frac{g}{2} t^2$ = about 40 m (131 ft.). In this case the minimum altitude is therefore about 50 m (164 ft.).

For the giant parachute with like initial speed (example 2) and an opening time $t = 5.8$ sec. the free falling distance is h = about 160 m (525 ft.) and the minimum utilization altitude is about 180 m (590 ft.). For the same parachute with an initial speed of 70.7 m/s (232 ft./sec.) (example 3), the opening time is $t = 4.1$ sec., corresponding to a free falling distance of 80 m

(262 ft.) and a minimum utilization altitude of about 100 m (328 ft.). These altitudes would be somewhat diminished by taking the air resistance into account. The comparison of the results with one another and with vertical descent plainly shows, however, the effect of the initial speed and its direction on the utilization altitude.

Translation by Dwight M. Miner,
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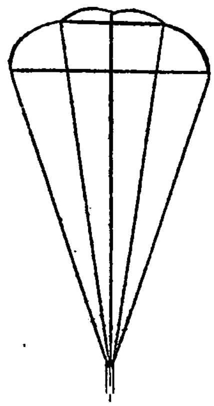


Fig.1 Parachute with secondary shroud lines.

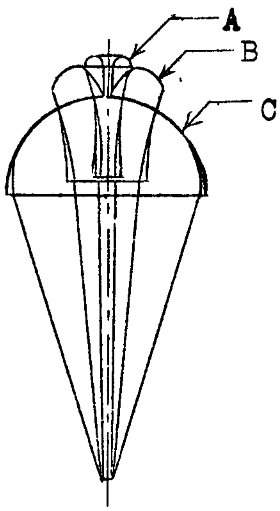
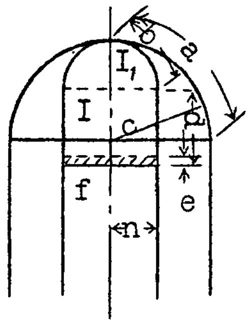


Fig.2 Opening shapes of normal hemispherical parachute with central shroud line.

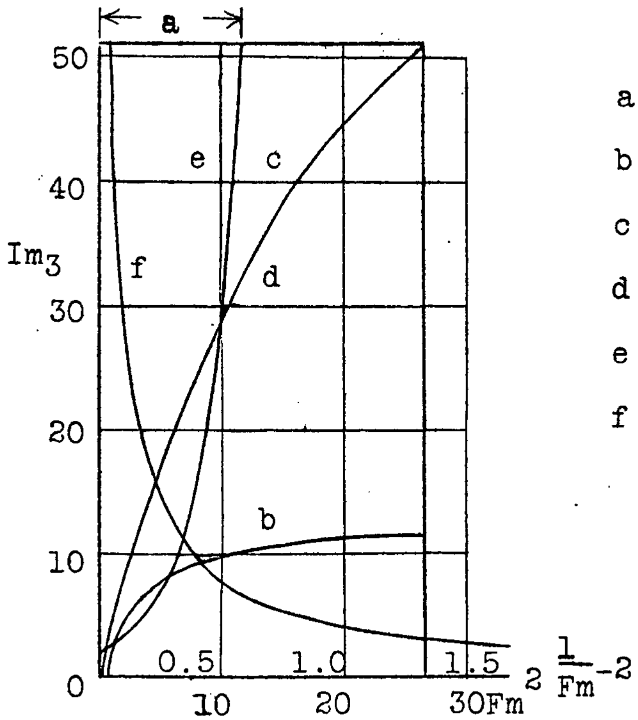


$a = r\,\frac{\pi}{2}$ $\qquad d = (r-n)\frac{\pi}{2}$

$b = n.\frac{\pi}{2}$ $\qquad e = \Delta s$

$c = I_2 r$ $\qquad f = \Delta I$

Fig.3 Opening shapes assumed as the calculation basis



$a = s = 11.2m$

$b = s = \psi(F)\ 1cm = 2m$

$c = I = f(F)\ 1cm = 2m^3$

$d = F = \varphi(I)\ 1cm = 2m^2$

$e = s = \phi(I)\ 1cm = 2m$

$f = \frac{1}{F} = \frac{1}{\varphi(I)} = \psi(I)\ 1cm = 0.1m^{-2}$

Fig.4 Inflation diagram for the opening process of a hemispherical parachute with the radius r=2.9m.

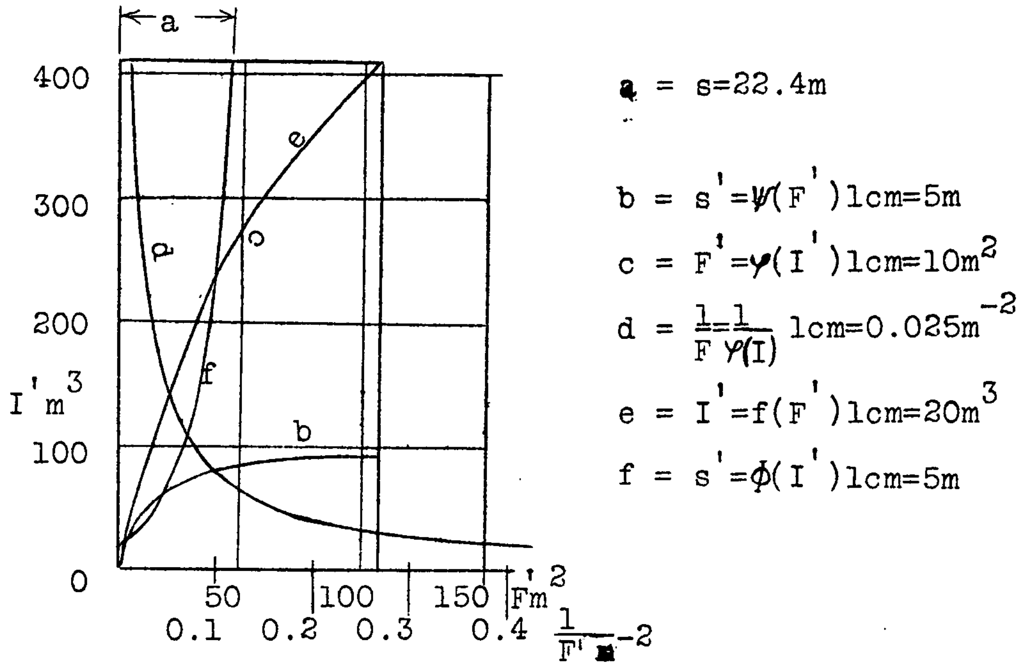


Fig.5 Inflation diagram for the opening process of a hemispherical parachute with the radius r=5.8m.

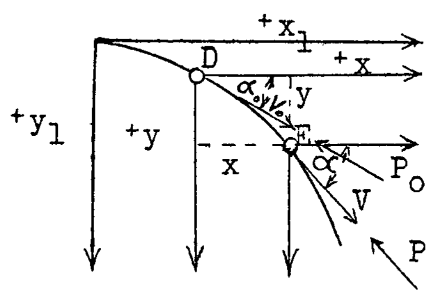


Fig.6 Path of parachute while opening.

$a = V' = \varphi(s')$

$b = V = \varphi(s)$

$c = \dfrac{P'_o}{m'} = f(s')$

$d = \dfrac{P_o}{m} = f(s)$

1 = example 1
2 " " 2
3 " " 3
4 " " 4
5 " " 5

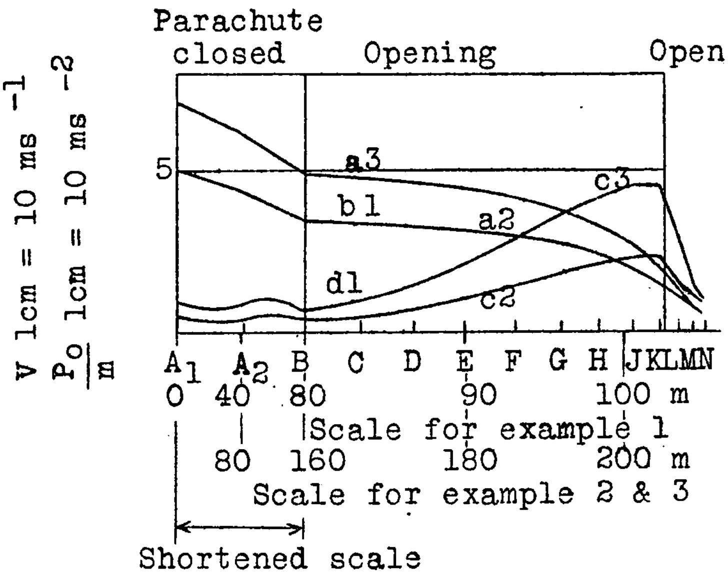
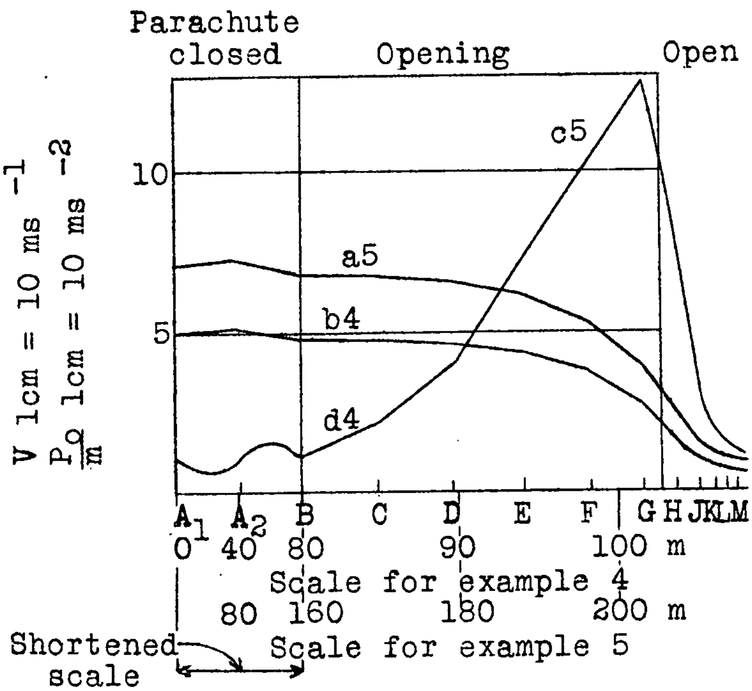
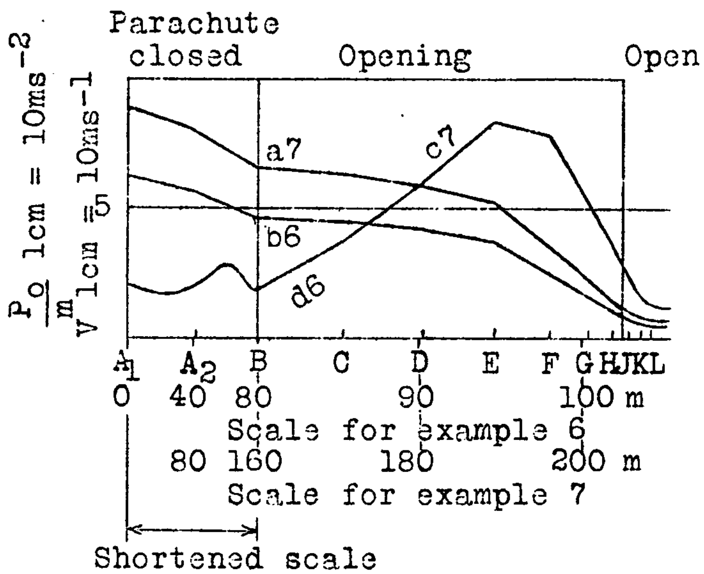


Fig.7 Stresses and speeds during opening process plotted against the horizontal distance.



Key for curves same as one above.

Fig.8 Stresses and speeds during opening process plotted against the vertical distance.



$a = V' = \varphi(s')$

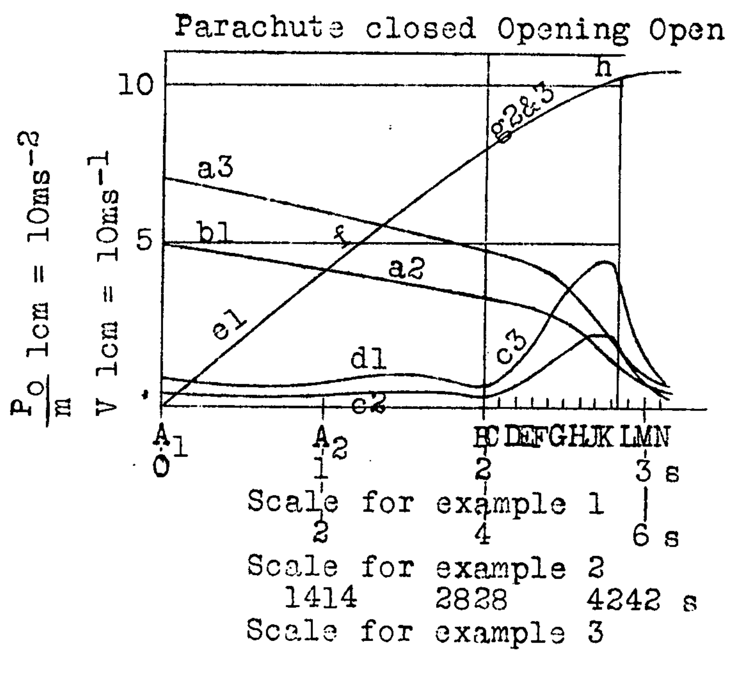
$b = V = \varphi(s)$

$c = \frac{P'_o}{m'} = f(s')$

$d = \frac{P_o}{m} = f(s)$

6 = example 6
7 " " 7

Fig.9 Stresses and speeds during opening process plotted against the vertical distance.



$a = V' = \varphi(t')$

$b = V = \varphi(t)$

$c = \frac{P'_o}{m'} = f(t')$

$d = \frac{P_o}{m} = f(t)$

$e = s = f(t)$

$f = 1cm = 10m$

$g = s' = f(t')$

$h = 1cm = 20m$

1 = example 1
2 " " 2
3 " " 3

Fig.10 Stresses, speeds and horizontal distances during opening process plotted against the time.

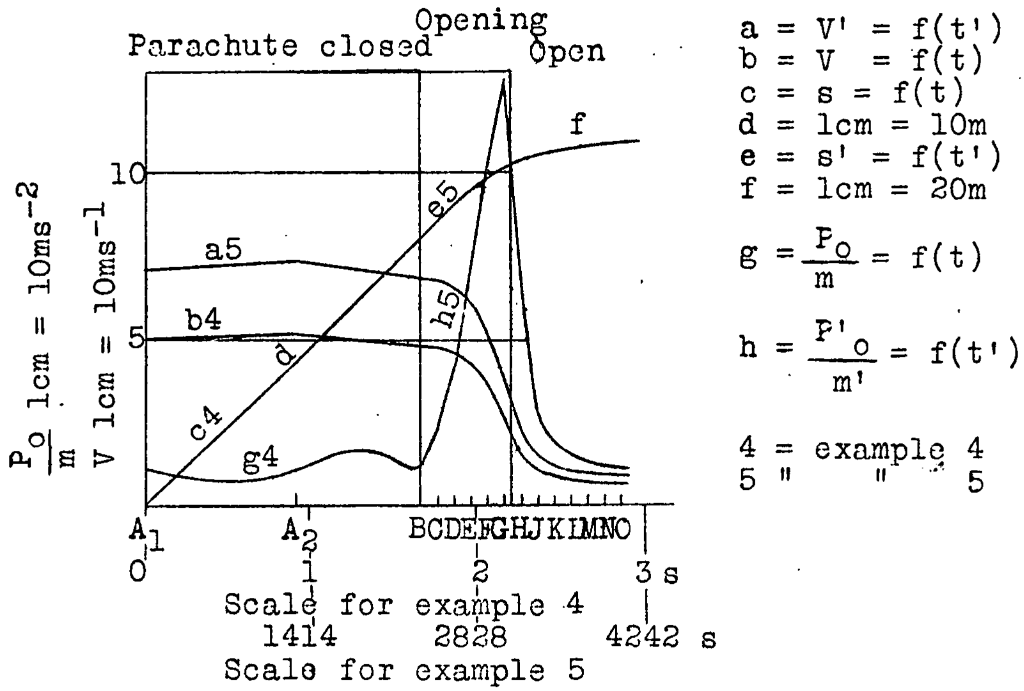


Fig.11 Stresses, speeds and vertical distances during opening process plotted against the time.

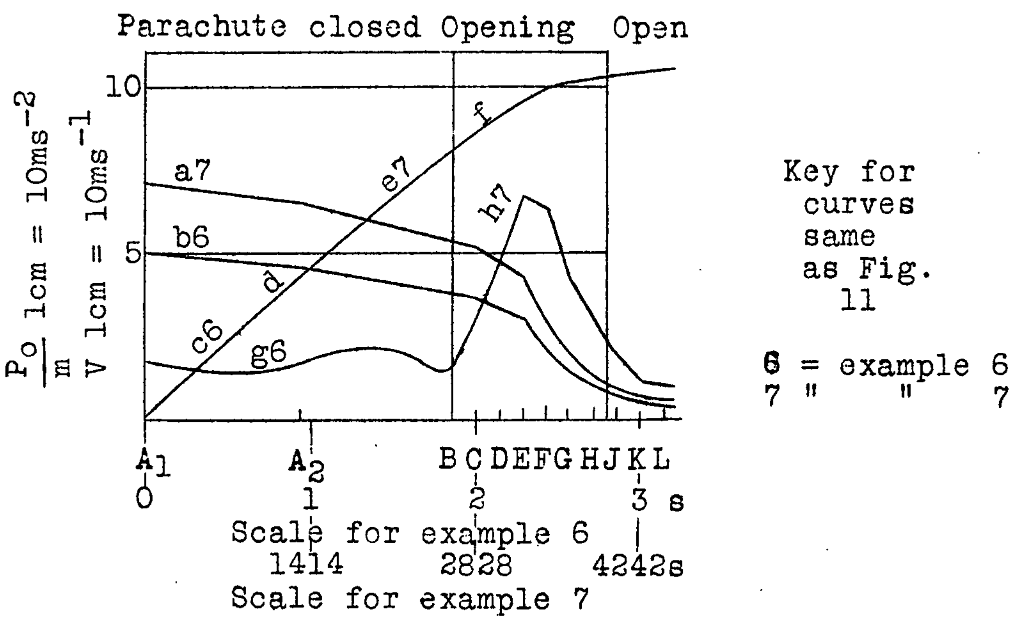


Fig.12 Stresses, speeds and vertical distances during opening process plotted against the time.